Analysis and Optimization of Contact Ratio of Asymmetric Gears

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Introduction

The contact ratio of spur gears is a critical parameter that affects gear drive performance. The influence of this parameter on the gear drive load capacity, efficiency, and noise and vibration is well known. There are studies (Refs. 1–3) dedicated to the analytical and experimental comparison of gears with low and high contact ratios. The dynamics and efficiency of high-contact-ratio asymmetric tooth gears were described in (Refs. 4–5).

These publications explore contact ratio using a very similar evaluation approach. The gears are designed traditionally, based on a preselected basic (or generating) rack. This makes the contact ratio dependent on the number of teeth of mating gears, basic rack addendum, and X-shifts. A contact ratio is considered nominal, as it is designed without influence of deflections under the operating load. Comparable gear sets with different contact ratios are identical in numbers of teeth, tooth size, and modules.

Such comparisons might have some theoretical value, but for practical gear design, equalizing some performance parameters in comparable gears is more important. For example, high-contact ratio gears provide load sharing between two or three pairs of teeth, increasing the load capacity. However, when they are compared with high-pressure angle and low-contact ratio gears (assuming identical numbers of teeth and tooth sizes), the mesh efficiency of high-contact ratio gears is significantly lower, because of their long tooth addendums and low pressure angle. Now a gear designer faces a dilemma: what is more important, high load capacity or high gear efficiency? Comparing gear sets with identical numbers of teeth and tooth size shows that it is impossible to simultaneously maximize both of these performance factors.

This article presents an analysis of asymmetric tooth gears considering the effective contact ratio that is also affected by bending and contact tooth deflections. The goal is to find an optimal solution for high performance gear drives, which would combine high load capacity and efficiency, as well as low transmission error (which affects gear noise and vibration).

Effective Contact Ratio and Transmission Error

The (trademarked) Direct Gear Design method (Ref. 6) defines the nominal contact ratio for external gears as:

$$\varepsilon_\alpha = \frac{z_1}{2\pi}(\tan\alpha_{a1} + u\tan\alpha_{a2} - (1+u)\tan\alpha_w) \tag{1}$$

where:

a_w = Operating pressure angle

α_{a1} and α_{a2} = Outer diameter profile angles

$u = z_2/z_1$ = Gear ratio

z_1 and z_2 = Number of teeth of mating pinion and gear

Effective contact ratio can be defined as the ratio of the tooth engagement angle to the angular pitch. The tooth engagement angle is a gear rotation angle from the start of the tooth engagement with the mating gear tooth to the end of the engagement. The effective contact ratio is:

$$\varepsilon_\alpha = \frac{\varphi_1}{360/z_1} = \frac{\varphi_2}{360/z_2} \tag{2}$$

where:

φ_1 and φ_2 = Pinion and gear engagement angles

$360/z_1$ and $360/z_2$ = Pinion and gear angular pitches

Transmission error is (Ref. 7) —

$$TE = r_{b2}(\theta_2 - u\theta_1) \tag{3}$$

where:

θ_1 and θ_2 = Driving pinion and driven gear rotation angles

r_{b2} = Driven gear base radius

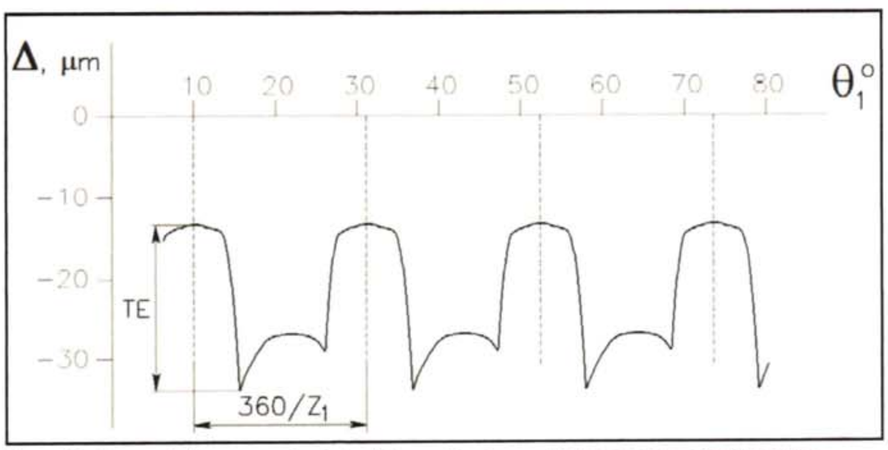

Figure 1 Transmission error chart; Δ – distance in microns between actual tooth contact point and ideal contact point.

A typical spur gear transmission error chart is shown (Fig. 1).

The effective contact ratio and transmission error are influenced by manufacturing tolerances and operating conditions, including deflections under load, temperature, etc. of the gears and other gearbox components. In this article, only bending and contact tooth deflections are considered for the definition of the effective contact ratio and transmission error. Each angular position of the driven gear relative to the driving gear is iteratively defined by equalizing the sum of the tooth contact load moments of each gear to its applied torque. The related

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tooth contact loads are also iteratively defined to conform to tooth bending and contact deflections, where the tooth bending deflection in each contact point is determined based on the FEA-calculated flexibility and the tooth contact deflection is calculated by the Hertz equation.

Comparable Gear Analysis

Comparable gear set macro geometry is defined by the Direct Gear Design method (Ref. 6); it allows for having the drive flank nominal contact ratio as one of the gear design input parameters. The mating gears have optimized root fillets. The specific sliding velocities are equalized to maximize gear mesh efficiency, which for external spur gears is equal to (Ref. 7):

$$E = 100 \times \left(1 - \frac{f}{2\cos\alpha_w} \times \frac{H_s^2 + H_t^2}{H_s + H_t}\right)\% \quad (4)$$

where:

f = Average friction coefficient

H_s = Specific sliding velocity at start of approach action

$$H_s = (1+u) \times \cos\alpha_w \times (\tan\alpha_{a2} - \tan\alpha_w) \quad (5)$$

H_t = Specific sliding velocity at end of recess action

$$H_t = (1+u) \times \cos\alpha_w \times (\tan\alpha_{a1} - \tan\alpha_w)/u \quad (6)$$

Maximum gear mesh efficiency is achieved when the specific sliding velocities $H_s = H_t$ are equalized. Then maximum mesh efficiency for external spur gears can be defined from Equations 4–6 (considering also Eq. 1) as:

$$E = 100 \times \left(1 - \frac{f\pi(1+u)}{2u} \times \frac{\varepsilon_\alpha}{z_1}\right)\% \quad (7)$$

All comparable gear sets are assumed to have identical maximized mesh efficiency E, average friction coefficient f, and gear ratio u. Then, according to Equation 7, the nominal contact ratio is inversely proportional to the pinion's number of teeth, as in:

$$\frac{\varepsilon_\alpha}{z_1} = \left(1 - \frac{E}{100}\right) \frac{2u}{f\pi(1+u)} \quad const. \quad (8)$$

The criterion 8 is used to analyze parameters of external spur gear sets with asymmetric teeth. Comparable asymmetric tooth gear sets have different numbers of teeth and identical center distance a_w, gear ratio u, coast flank pressure angle α_{wc}, minimal pinion and gear tooth tip thicknesses th_{a1} and th_{a2} that are required to avoid the harden through tooth tips for the carburized harden gears, average friction coefficient f and gear mesh efficiency E, pinion and gear material properties, and equalized specific sliding velocities H_s and H_t. The face widths b_1 and b_2 are defined to approximately equalize the pinion and gear tooth bending stresses considering the optimized root fillets.

If the center distance is identical for all gear sets, the operating modules are inversely proportional to the number of pinion teeth and defined as:

$$m = \frac{2a_w}{z_1(1+u)} \quad (9)$$

The operating pitch diameter tooth thickness ratio:

$$TTR = S_{w1}/p_w = S_{w1}/(S_{w1} + S_{w2}), \quad (10)$$

where:

S_{w1} and S_{w2} = Pinion and gear tooth thicknesses at the operating pitch diameters

p_w = Operating circular pitch

The operating pitch diameter tooth thickness ratio value is TTR selected to provide equalized specific sliding velocities H_s and H_t and identical pinion and gear tooth tip thicknesses th_{a1} and th_{a2}.

The maximized drive flank pressure angle α_{wd} is defined to achieve minimal contact stress. It must also provide the nominal drive contact ratio $\varepsilon_{\alpha d}$ defined by Equation 1, the preselected values of the coast flank pressure angle α_{wc}, and pinion and gear tooth tip thicknesses th_{a1} and th_{a2}.

The asymmetry factor is:

$$K = \cos\alpha_{wc}/\cos\alpha_{wd} \quad (11)$$

The bearing load is:

$$F = 2000T_1/d_{bd1}, \quad (12)$$

where:

T_1 = Pinion operating torque in Nm

d_{bd1} = Pinion drive flank base diameter in mm

Load sharing factor is:

$$L = F_{cmax}/F, \quad (13)$$

where:

F_{cmax} = Maximum contact load in the single tooth set contact

Table 1 Gear parameters for several gear sets defined to satisfy pre-selected comparison conditions

Gear set comparison conditions		Center distance – 150 mm; Gear ratio – 2:1; Coast Pressure Angle – 15°; Pinion and Gear face widths – 35 mm and 30 mm; Tooth tip thickness – 0.30-module; Average friction coefficient – 0.05; Gear mesh efficiency – 99%; Pinion and Gear material properties: Modulus of elasticity – 207,000 MPa, Poisson ratio – 0.3; Pinion Torque – 1500 Nm; All gears have optimized tooth root fillets.														
Numbers of teeth	Pinion	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
	Gear	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56
Module, mm		7.143	6.667	6.25	5.882	5.556	5.263	5.000	4.762	4.545	4.348	4.167	4.000	3.846	3.704	3.571
Tooth Thickness Ratio		0.518	0.520	0.522	0.524	0.526	0.527	0.529	0.531	0.532	0.534	0.536	0.537	0.538	0.540	0.542
Drive Pressure Angle, °		42.0	39.1	36.6	34.5	32.7	31.1	29.7	28.3	27.0	25.8	24.8	23.9	23.0	22.3	21.5
Asymmetry Factor		1.300	1.245	1.203	1.172	1.148	1.128	1.112	1.097	1.084	1.073	1.064	1.057	1.049	1.044	1.038
Nominal Drive Contact Ratio		1.11	1.19	1.27	1.35	1.43	1.51	1.59	1.67	1.75	1.83	1.91	1.99	2.07	2.15	2.23
Effective Drive Contact Ratio		1.24	1.33	1.42	1.51	1.60	1.69	1.78	1.86	1.96	2.04	2.13	2.22	2.31	2.40	2.49
Specific Sliding Velocities		0.279	0.292	0.301	0.309	0.316	0.321	0.322	0.330	0.334	0.338	0.340	0.343	0.345	0.347	0.349
Bearing Load, N		40368	38601	37417	36448	35647	35037	34890	34072	33673	33320	33045	32839	32616	32422	32247
Load Sharing Factor		1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.743	0.692	0.674	0.644	0.630	0.620
Contact Ratio Type		Low					Medium				Transitional			High		
Selected Gear Sets		-	**1**	-	-	-	**2**	-	-	-	**3**	-	-	-	**4**	-

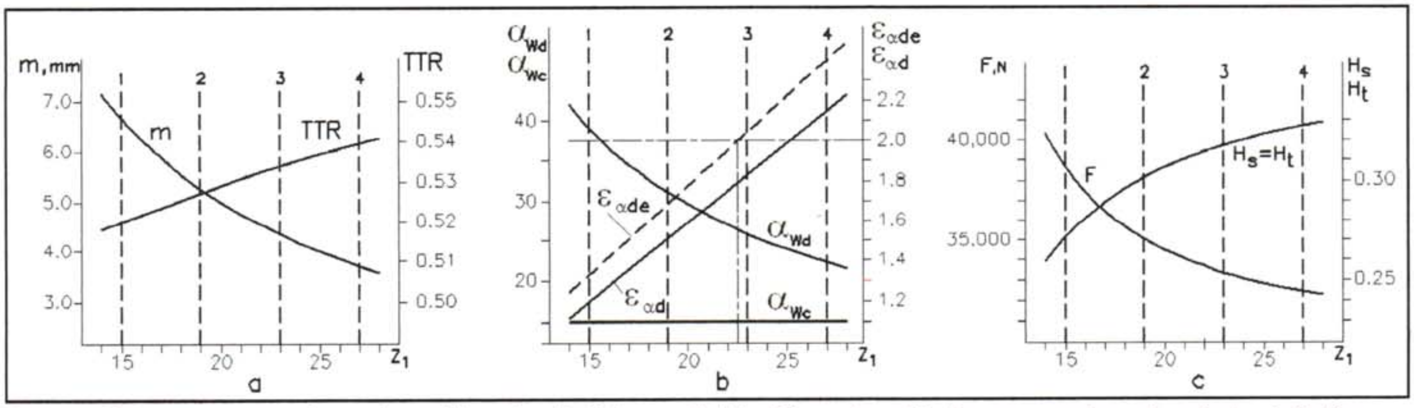

Figure 2 Main gear parameter charts; a)—module and tooth thickness ratio; b)—drive and coast flank pressure angles, and nominal and effective drive contact ratios; c)—bearing load and specific sliding velocities.

If the drive flank effective contact ratio $\varepsilon_{\alpha de} < 2.0$, the load sharing factor $L = 1.0$. Similar to the effective contact ratio and transmission error, the load sharing factor is defined accounting only for the bending and contact tooth deflections.

- Table 1 presents gear parameters for several gear sets that are defined to satisfy pre-selected comparison conditions. The highlighted parameters for four gear sets are selected to define the transmission error under variable operating loads and to find a gear set with the optimal contact ratio.
- Gear set 1 has a 15-tooth pinion and 30-tooth gear with a low contact ratio ($\varepsilon_{\alpha d} = 1.19$ and $\varepsilon_{\alpha de} = 1.33$.
- Gear set 2 has a 19-tooth pinion and 38-tooth gear with a medium contact ratio ($\varepsilon_{\alpha d} = 1.51$ and $\varepsilon_{\alpha de} = 1.69$).
- Gear set 3 has a 23-tooth pinion and 46-tooth gear with a transitional contact ratio ($\varepsilon_{\alpha d} = 1.83$ and $\varepsilon_{\alpha de} = 2.04$). It is called transitional because it has a nominal contact ratio < 2.0 and an effective contact ratio under the given operating load > 2.0. Such gear sets under low load have one or two mating tooth pairs in contact. When the load is increased to its operating level and tooth deflections are increased, the gears are engaged in two or three mating tooth pairs in contact. These results in tooth load sharing and a single-tooth load reduction.
- Gear set 4 has a 27-tooth pinion and 54-tooth gear with a high contact ratio ($\varepsilon_{\alpha d} = 2.15$ and $\varepsilon_{\alpha de} = 2.40$).

The main gear parameters vs. pinion number of teeth charts are shown (Fig. 2).

The Figure 2 charts indicate that with increasing numbers of pinion teeth, the tooth thickness ratio TTR also increases slightly, the drive flank pressure angle α_{wd} lowers, but the nominal and effective contact ratios $\varepsilon_{\alpha d}$ and $\varepsilon_{\alpha de}$ grow. As a result of

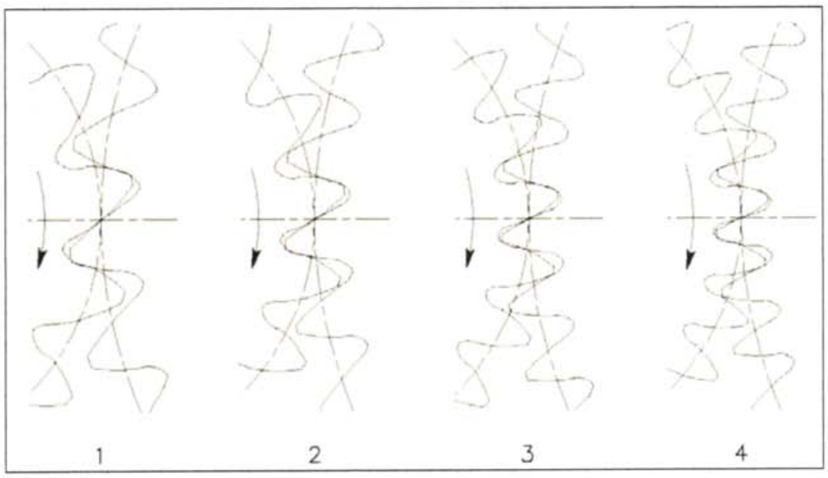

Figure 3 Selected gear sets.

Table 2 Results of selected gear set analysis under different driving torques

Gear set 1 – low contact ratio: $\varepsilon_{ad} = 1.194$, $z_1 = 15$, $z_2 = 30$, m = 6.667 mm, $\alpha_{wd} = 39.0°$, $\alpha_{wc} = 15.0°$								
Pinion Torque, Nm	250	500	750	1000	1250	1500	1750	2000
Contact Stress, MPa	714	921	1115	1288	1433	1570	1695	1812
Pinion Bending Stress, MPa	53.7	108	161	215	269	322	376	430
Gear Bending Stress, MPa	52.9	106	159	212	264	317	370	423
Effective Drive Contact Ratio	1.25	1.28	1.29	1.31	1.32	1.33	1.34	1.35
Transmission Error, mm	3.1	6.1	8.8	11.3	13.7	16.1	18.4	20.8
Load Sharing Factor	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Gear set 2 – medium contact ratio: $\varepsilon_{ad} = 1.512$, $z_1 = 19$, $z_2 = 38$, m = 5.263 mm, $\alpha_{wd} = 31.1°$, $\alpha_{wc} = 15.0°$								
Pinion Torque, Nm	250	500	750	1000	1250	1500	1750	2000
Contact Stress, MPa	717	930	1132	1311	1464	1601	1726	1845
Pinion Bending Stress, MPa	65.0	130	195	260	325	390	455	520
Gear Bending Stress, MPa	66.9	134	201	267	334	401	468	535
Effective Drive Contact Ratio	1.59	1.63	1.65	1.67	1.68	1.69	1.72	1.74
Transmission Error, mm	2.9	5.5	8.2	10.6	12.9	15.1	17.2	19.3
Load Sharing Factor	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Gear set 3 – transitional contact ratio: $\varepsilon_{ad} = 1.831$, $z_1 = 23$, $z_2 = 46$, m = 4.348 mm, $\alpha_{wd} = 25.8°$, $\alpha_{wc} = 15.0°$								
Pinion Torque, Nm	250	500	750	1000	1250	1500	1750	2000
Contact Stress, MPa	740	962	1143	1260	1348	1436	1528	1622
Pinion Bending Stress, MPa	76.8	153	216	258	301	353	412	470
Gear Bending Stress, MPa	81.4	162	228	271	314	359	408	458
Effective Drive Contact Ratio	1.95	1.99	2.01	2.02	2.03	2.04	2.06	2.08
Transmission Error, mm	2.6	5.0	7.3	6.8	6.2	5.8	5.9	6.3
Load Sharing Factor	1.0	1.0	0.944	0.848	0.783	0.748	0.731	0.722
Gear set 4 – high contact ratio: $\varepsilon_{ad} = 2.149$, $z_1 = 27$, $z_2 = 54$, m = 3.704 mm, $\alpha_{wd} = 22.3°$, $\alpha_{wc} = 15.0°$								
Pinion Torque, Nm	250	500	750	1000	1250	1500	1750	2000
Contact Stress, MPa	661	820	974	1121	1253	1366	1470	1572
Pinion Bending Stress, MPa	70.0	140	210	280	350	420	490	560
Gear Bending Stress, MPa	65.2	130	196	261	326	392	457	522
Effective Drive Contact Ratio	2.25	2.3	2.33	2.36	2.38	2.40	2.42	2.44
Transmission Error, mm	1.5	2.9	4.2	5.5	6.7	7.8	8.9	9.9
Load Sharing Factor	0.655	0.648	0.643	0.639	0.636	0.634	0.632	0.630

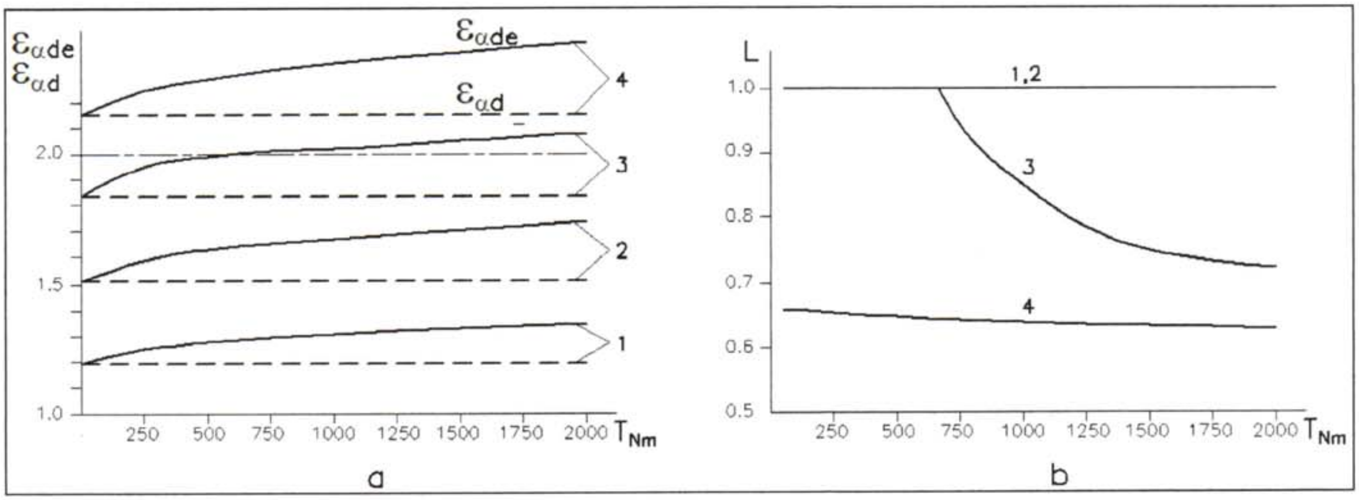

Figure 4 a) nominal and effective contact ratios; b) load sharing factor.

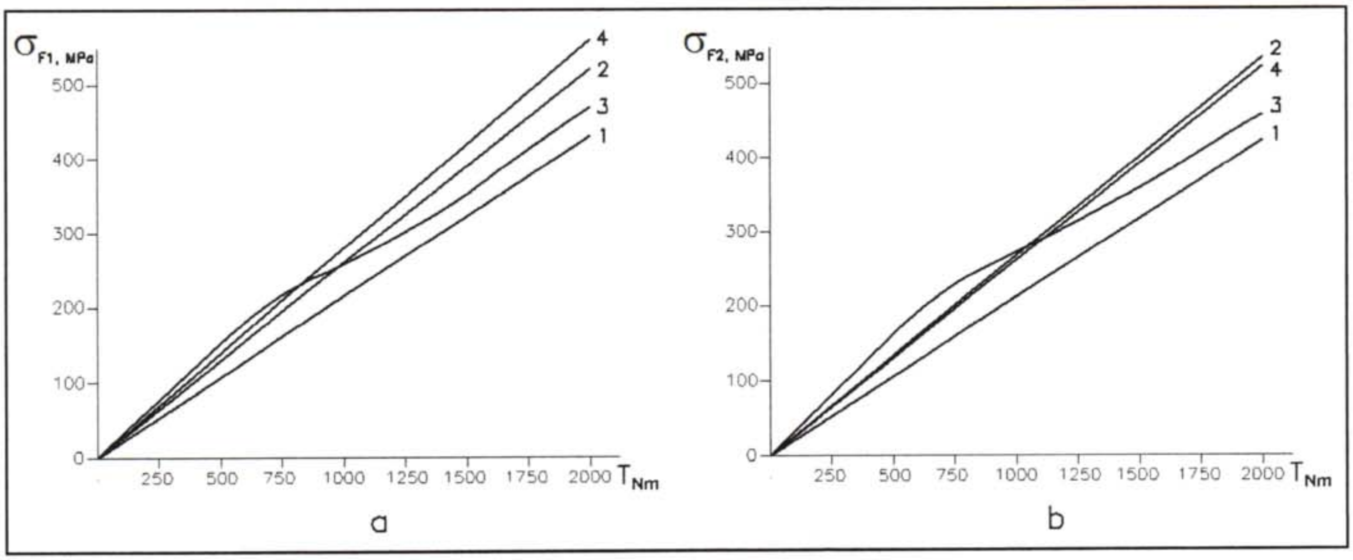

Figure 5 Pinion a) and gear b) bending stress charts.

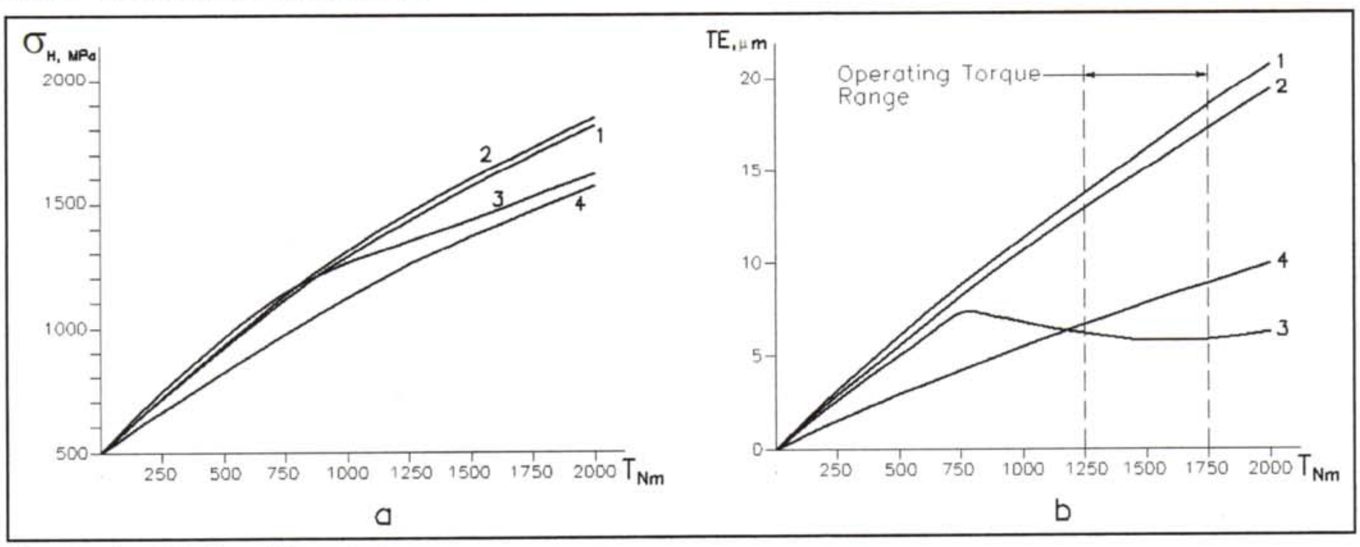

Figure 6 a) contact stress; b) transmission error.

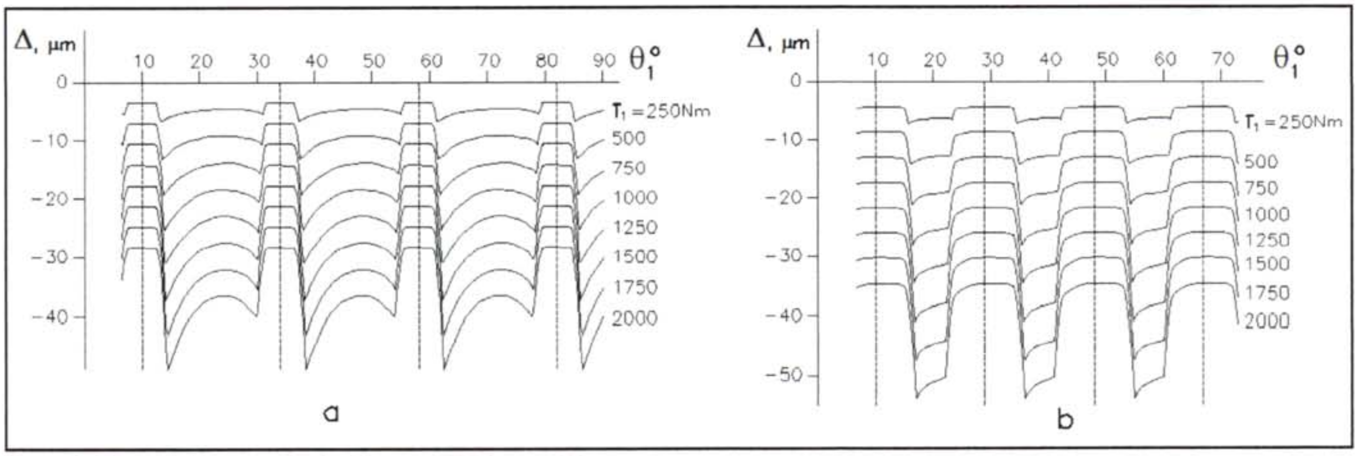

Figure 7 **Transmission error charts: a) gear set 1; b) gear set 2.**

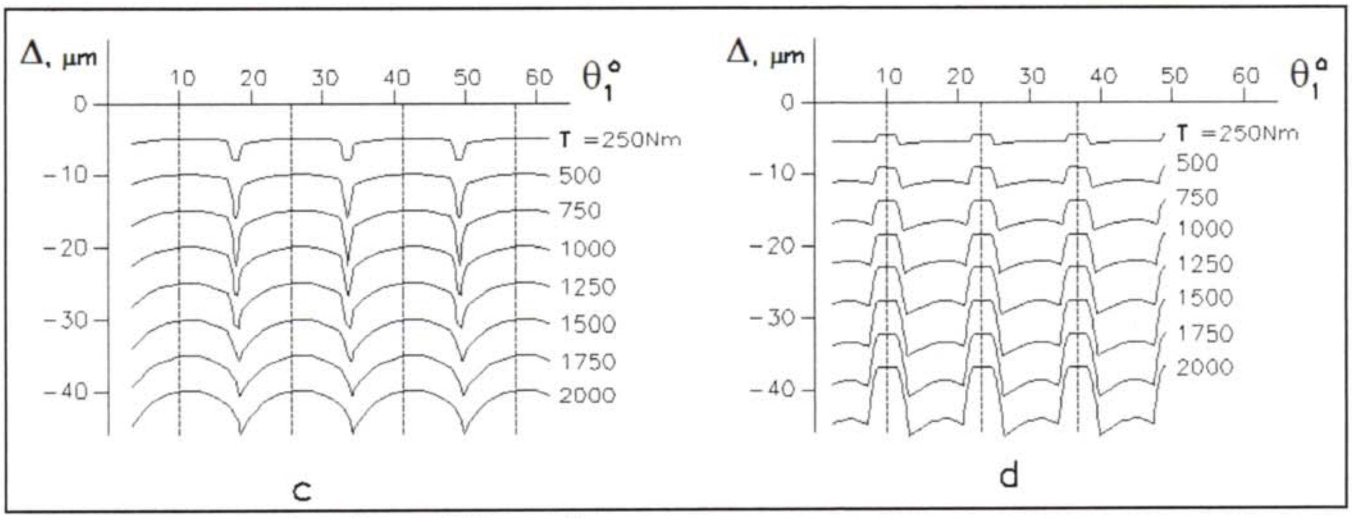

Figure 8 **Transmission error charts: a) gear set 3; b) gear set 4.**

the drive flank pressure angle reduction, the bearing load F is noticeably reduced, but the equalized specific sliding velocities H_s and H_t are increased because of the increased contact ratios.

Figure 3 shows the selected gear set meshes at the same scale; arrows indicate the driving pinion torque direction.

Results of the selected gear sets analysis under different driving torques are shown in the Table 2.

Main gear parameters vs. pinion operating torque for gear sets 1–4 are shown in Figs. 4–6.

Figures 7 and 8 present the transmission error charts of gear sets 1–4 at different driving torques.

The charts in Figure 6b clearly indicate that with increasing operating torque, the transmission error of gear sets 1, 2, and 4 increases as well. In gear set 3 the transmission also increases until the effective contact ratio exceeds 2.0 and the gear engagement is converted from 1–2 mating tooth pair contact to the 2–3 mating tooth pair contact. Then the transmission error of gear set 3 decreases slightly, stays flat, and then gradually increases. Within the operating torque range, gear set 3's transmission error is the lowest in comparison to the other gear sets.

Summary

The article presents an analysis of nominal and effective contact ratios of several sets of spur asymmetric tooth gears with equal maximized gear mesh efficiencies but different numbers of teeth. This analysis has defined the main gear performance parameters, including tooth bending and contact stresses and transmission errors under variable operating load, accounting for bending and contact stress deflection. It demonstrated that transitional contact ratio gears appeared to be an optimal solution within the operating load range, providing minimal transmission error, bending stress that is lower than that of gear sets with medium and high contact ratio gears, and contact stress that is lower than that of gear sets with low and medium contact ratio gears. These transitional contact ratio gears with relatively constant transmission error within operating load range are potentially good for tooth flank microgeometry optimization for additional transmission error reduction. This analysis confirms the article (Ref. 3) conclusion that gears with integer values for the contact ratio are inherently quiet, when the effective contact ratio is considered instead the nominal contact ratio.

The presented contact ratio analysis and optimization method is equally applicable and can be very beneficial for directly designed symmetric tooth gears.

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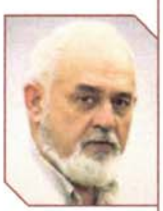

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